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How do Prospective Teachers Solve Routine and Non-Routine Trigonometry Problems?

Sefa Dündar[1] and Hakan Yaman[2]

[1,2]*Abant Izzet Baysal University, Education Faculty, Mathematics Education, Bolu- Turkey.*

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ABSTRACT

The aim of this study is to identify prospective teachers' skills in transferring their trigonometry knowledge to solve problems they encounter. Test on Solving Routine Problems (TSRP) and Test on Solving Non-routine Problems (TSNRP), were used as data collection tools. As a result of the data analysis the prospective teachers correctly answered nearly all questions in the TSRP that required procedural knowledge, whereas they experienced problems in TSNRP that required their skills of transferring their trigonometry knowledge. In view of these results, it was observed that the prospective teachers were able to answer procedural questions with their trigonometry knowledge.

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Keywords:
Trigonometry, routine problems, non-routine problems, mathematics education

Introduction

The general principles and standards of school mathematics are explained in the document entitled as Principles and Standards of School Mathematics (PSSM) issued by an American-based institution, National Council of Teachers of Mathematics (NCTM), for the 6th-8th graders. These standards consist of content standards and process standards. Problem solving, reasoning and proof, communication, connections, and representation are featured as the process standards in this document. Usage of these standards is particularly recommended throughout the whole process in courses. When the Elementary Mathematics Course Teaching Program in Turkey was examined, it was stated that the instruction should have aims to improve mathematical concepts as well as certain skills, such as reasoning, associating, communication and problem solving. However, problem solving is one of the most important skills (TME, 2009; 8).

To improve the problem-solving skills is a prominent consideration among the objectives of elementary mathematics course. Altun (2005) explains the importance of problem solving in mathematics teaching as follows: "It is to provide individuals with the mathematical information and skills that are required by daily life, in general, in order to teach them how to solve problems and to provide them with a way of thinking that deals with situations within a problem solving approach."

A problem must be defined well in order to define problem solving in the beginning Umay (2007) defines the problem as a situation in which the solution is not seen clearly and which requires the solver to use his/her mind in contemplation of a solution by drawing on their own experiences. According to Olkun and Toluk-Uçar (2012), problem solving involves situations that evoke in the individual a desire to solve the problem. These situations have no available solution procedures, but the individuals can solve them using their own knowledge and experience. In accordance with the concept of problem, problem solving can be defined as "Knowing what must be done in situations where one does not know what to do" (Çelebioğlu & Yazgan, 2009). Since problem solving is also a scientific method, it requires critical thinking, creative and

[1] Corresponding author's address: Abant Izzet Baysal University, Education Faculty, Mathematics Education, Bolu-Turkey
Telephone: 0 374 254 10 00 - 1617
e-mail: sefadundar@gmail.com
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reflective thinking, analysis and synthesis skills (Soylu & Soylu, 2006). According to Işık and Kar (2011), the problem solving skill is a skill that assists students in both solving the problems that they will come across and drawing the connection between mathematics and real-life situations.

Understanding mathematical knowledge and drawing the relationship between these pieces of knowledge stands out in the process of problem solving (Swing & Peterson, 1988). Students must combine the concepts and the procedures and apply them to the solution of the problem during problem solving (Bernardo, 1999). Conceptual knowledge does not only consist of recognizing the concept or knowing the definition and the name of the concept, but at the same time, being able to see the mutual transitions and relationships among the concepts. Procedural knowledge can be explained in terms of the two separate sections that form it. The first section of the procedural knowledge involves the symbols and language of mathematics whereas the second section involves the relations used in solving mathematics problems, procedures on concrete objects, visual diagrams, imagining or other nonstandard objects of the mathematics system (Hiebert & Lefevre, 1986). When procedural knowledge or knowledge of rules is combined with conceptual knowledge, the individual can explain not only how the procedures are performed, but also why they are performed. Failure to gain the conceptual basics of procedural knowledge and draw a relationship between procedural knowledge and concepts results in a failure to build the models and decide where the procedures will be used. This manifests itself as a failure in problem solving (Baykul, 2005). It is observed that there is an important relationship between conceptual knowledge and procedural knowledge.

Mathematical knowledge can be learned by balancing procedural knowledge and conceptual knowledge (Baki, 1998; Van de Walle, 2004). When mathematics courses are not given predominantly conceptually, an inclination towards memorization will emerge instead of learning. Generally, procedures are given importance instead of the concepts in teaching since conceptual knowledge is gained by memorization only as a rule without researching the reasons and causes (Baki & Kartal, 2004). In this process, many students are not aware of the fact that there are concepts at the basis of the procedures they use, and they do not know what mathematics means. They believe that learning mathematics is to perform procedures on meaningless symbols, and they try to learn mathematics by memorization (Oaks, 1990). The student is a good mirror in learning by memorization. The student masterfully reflects what is given to him/her, but he/she does not produce anything. Regarding conceptual learning, the student is a problem solver who can effectively use his/her own creativity, intuitions and skills in solving problems and producing mathematical knowledge. For this reason, the conceptual learning approach regards mathematics as a network of interconnected concepts and thoughts, and advises students to structure mathematical concepts and thoughts by themselves instead of copying them from outside (Bell & Baki, 1997). To learn mathematics is not only to fill the mind with available information, but also to use that information in a way that enables us to manifest our own thoughts (Baki & Kartal, 2004). When solving a problem, an individual must undergo a cognitive process that involves understanding the problem sentence; making a plan for the solution; applying the plan and making an evaluation.

The conducted researches set forth that problem solving instruction in schools fell short in solving real-life problems and students resorted to numerical procedures and strived to reach a solution quickly instead of contemplating the problems and generating solution strategies (Altun, Memnun, & Yazgan, 2007; Cai, 2003; De Corte, 2004; Kaur, 2001; Pape & Wang, 2003; Nancarrow, 2004; Verschaffel, et al., 1999).

In their study, Altun and Arslan (2006) give two basic reasons for students' failure in solving the problems that they come across in real life:

(i) Insufficient field knowledge: mathematical symbols, formulas, misconceptions, etc.

(ii) Hardships regarding creativity, doing on purpose and being aware of what one is doing

On the other hand, we come across types of problems in the problem solving process. When the researchers conducted on the types of problems are examined, it is observed that there is a distinction between routine problems and non-routine problems. According to Altun (2005), non-routine problems require more thinking compared to the routine ones and in which the method used to solve the problem is not clear. According to Artut and Tarım (2009), routine problems are generally similar examples of a previously solved problem or they require applying a learned formula to a new situation. Researchers

criticize the fact that only routine problems are used in instruction (Özdoğan & Kula, 2007). Non-routine problems require not only mathematical thinking but also skills such as reasoning in order for the students to find a different algorithm than the one they learned in the classroom (Işık & Kar, 2011).Research conducted indicates that many of the students and prospective teachers are not good at solving non-routine mathematics problems (Altun ve Arslan, 2006; Dündar (in press); Higgins, 1997; Verschaffel, et al., 1999; Holton, Anderson, Thomas, & Fletcher, 1999). It is also seen in many research examining problem solving adequacies that students had problems in solving non-routine problems (Verschaffel et al., 1999). When these difficulties were examined, it appeared that the difficulties were related with inadequacies in using and understanding of concepts, formulas, relations and algorithms together with thought in the process of problem solving (Arslan & Altun, 2007; Bayazit, 2013).

The concept of trigonometry, which is used in not only fields such as medicine, physical sciences and economy but also on the foundation of other mathematical concepts, has many fields of application. Trigonometry was noticeably absent from this newer set of standards for the secondary classroom (NCTM, 2000). The only reference to the subject comes about in the representation standard where it is stated, "They (students) should recognize, for example, that phenomena with periodic features often are best modeled by trigonometric functions" (NCTM, 2000, p. 361). Trigonometry is especially used in limits, derivative and integral calculations. The fact that algorithmic (procedural) aspects of this concept are frequently studied causes its conceptual aspect to be neglected. Situations such as applying the formulas automatically without contemplating the problem and a failure to draw the relationship between trigonometry and other concepts due to this algorithmic approach can be listed among the reasons for the failure in forming conceptual knowledge. Trigonometry is an important concept in improving a student's reasoning ability (Kültür, Kaplan, & Kaplan, 2008). In this respect, real-life problems must be featured in teaching the subject of trigonometry and students must be enabled to gain skills such as interpretation and reasoning, associating and critical thinking.

When conducted studies were examined, it was seen that researchers studied on the ways of teaching and learning trigonometry better. In these studies, it was appeared that the effects of trigonometry knowledge, technology use or different methods on this knowledge (Choi-Koh, 2003; Dion, Harvey, Jackson, Klag, Liu, & Wrightt, 2001). Choi-Koh (2003) researched the effects of graphing calculators on trigonometry teaching by studying with one student. In this study, the student's understanding of trigonometry was focused. As a result of the study, it was found that the student could relate among different representation styles and problem solving skill of the student increased by means of graphing calculators. In the study of Brown (2005) conducted with high school students, it was aimed to reveal trigonometry knowledge of students. It appeared that the students had lack of knowledge about main points of trigonometry after 4-6 weekly trigonometry lessons. In the studies conducted with teacher candidates, it was found that they had lack of trigonometry knowledge similarly (Fi, 2003; Čižmešija, & Milin Šipuš, 2013; Topçu, Akkoç, Yılmaz & Önder, 2006, Nason, Chinnappan, & Lawson, 1996). When literature was examined, it was seen that there were studies investigating the effect of technology on trigonometry knowledge of teacher candidates which were similar to studies with students (Steckroth, 2007; Arcavi, 2003). In these studies, it appeared that visualization increased and trigonometry knowledge developed by means of technology.

Considering the fact that problem solving is one of the basic skills of mathematics instruction, it is important to learn problem solving skills and how to teach students these skills in an effective way. The focus of the studies conducted in our country is generally on the success of elementary and secondary school students in routine and non-routine problems as well as the solution strategies regarding routine and non-routine problems. However, it is very important to do a research on the degree to which prospective teachers, who will teach this student group, are successful in routine and non-routine problems and what type of mistakes they make.

The studies examining problem solving processes –especially non-routine problems- of prospective teachers who will teach mathematics to primary and secondary students (Altun et al., 2007; Arslan & Altun, 2007; Bayazit, 2013) can reveal useful information to eliminate the inadequacies of prospective teachers in this regard. The aim of this study was to reveal the success of prospective mathematics teachers in routine and non-routine problems using their trigonometry knowledge. Answers were sought to the following research questions within the scope of this aim.

1. Is there a significant difference between prospective teachers' performances in Test on Solving Routine Problems (TSRP) and Test on Solving Non-routine Problems (TSNRP)?
2. Is there a significant relationship between the scores of the prospective teachers in the questions of TSRP type and the questions of TSNRP type?
3. How do the prospective teachers explain the mistakes that they made in TSRP?
4. How do the prospective teachers explain the mistakes that they made in TSNRP?

Method

Research design, sample, data collection tools and the statistical methods and techniques used in data analysis were explained in this section.

Research Design

A mixed method composed of quantitative and qualitative techniques was used in this research. The quantitative data was analyzed at first, and then, the qualitative data was collected in order to both retrieve detailed information from the prospective teachers about this process and support other findings. The mixed method, which is conducted in this manner, is defined as the "explanatory design". The reason for selecting this design was the necessity to support the data, which was collected through the quantitative method. The stages of the explanatory design are shown in Figure 1 (Fraenkel & Wallen, 2006; Patton, 2002).

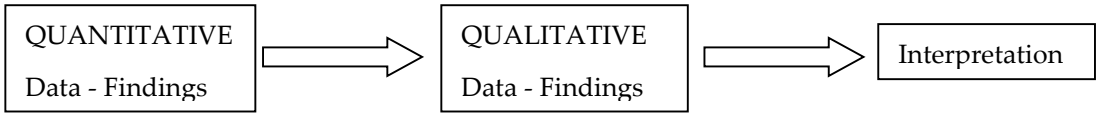


Figure 1: Stages of the explanatory design

In the first section of the research, quantitative data was collected from the participants in accordance with their trigonometry knowledge and statistical analyses, which were conducted in order to answer the identified research questions. In the second section, comprehensive interviews were conducted with all participants and qualitative data, which was collected in accordance with the results of these statistical analyses. Benefiting from this data, an attempt was made to set forth on how the prospective teachers used their trigonometry knowledge in routine and non-routine problem types.

The survey method was used in the quantitative section of the research since the aim was to reveal the existing situation and to depict it in the way that it existed. Furthermore, correlational method was used because the aim in this study was also to determine the existence and/or the degree of the change between two variables (Fraenkel & Wallen, 2006:358). The correlational method was used in studying the interrelations between prospective teachers' scores in the TSRP and the TSNRP.

In the qualitative section of the research, the reasons for the incorrect answers that were given by all prospective teachers to the questions on the tests and how they answered the questions were determined with "semi-structured interview technique" after the TSRP and the TSNRP had been completed.

Sample

The purposive sampling method was used as the sampling method (Fraenkel & Wallen, 2006). Fifty-three prospective teachers, who were first-year students at the Department of Elementary Mathematics Instruction at the faculty of education at a state university in Turkey, were selected as an example using the purposive sampling method. Gender-based distribution of the prospective teachers is given in Table 1.

Table 1: Gender-based distribution of the prospective teachers that participated in the research

Gender	Frequency	Percentage (%)
Male	11	20.8
Female	42	79.2
Total	53	100

No universe or sample was selected in the qualitative section of the research since it was not going to be possible to make a generalization. However, all prospective teachers (53 prospective teachers) participated in the interviews.

Collection of the Quantitative Data

Test on Solving Routine Problems (TSRP) and Test on Solving Non-routine Problems (TSNRP) were used as data collection tools in order to identify prospective teachers' skills to utilize their trigonometry knowledge to solve problems they may encounter. TSNRP is composed of five verbal problem situations which require to transfer a daily life situation to trigonometry. The questions in the TSRP were formed by transferring the problems in the TSNRP and non-verbal information, whereas the information used in the problem to the procedure. On the other hand, TSRP and TSNRP are composed of the same questions in numerical terms. The question was asked as a procedure in TSRP whereas it was put in a verbal problem situation in TSNRP (see Figure 2). Data obtained from TSRP and TSNRP was evaluated in "correct" and "incorrect" categories. The highest score that can be taken from these tests is 5 whereas the lowest score is 0. In this study, the answers have not been evaluated with partial credit model to ensure consistency in the scoring process.

(TSNRP) Question 2: The angle of sunrays that is formed with a shadow of a tree of 25 m is 30°. Find the height of the tree.

(TSRP) Question 2: |CB|=25m, s(BCA)=30°,

According to the given information, |AB|=?

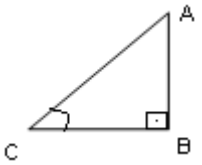


Figure 2: TSRP and TSNRP question types

The opinions of three field experts who were studying mathematics instruction, 1 assessment and evaluation expert were taken in order to establish the content validity of the prepared tests. Accuracy of the distinction of problems as routine and non-routine and clarity of verbal problems including non-routine problems were discussed in the framework of expert opinions. It was revealed whether the problems are understandable by being given back the problems to 5 teacher candidates studying in the first grade of the department of primary mathematics teacher education and provided that the construct validity of the test. The finalized tests were implemented on 62 first-year prospective mathematics teachers from another university. The pilot study was conducted within the same day in two different sessions. Following these implementations, KR-20 reliability coefficients were calculated in order to find the reliability of the measurements belonging to the tests. KR-20 results were found as .68 for the TSNRP and .67 for the TSRP. These results were considered appropriate for the reliability of the test measurements due to the fact that the number of questions was considered low.

Collection of the Qualitative Data

The answers given by the prospective teachers in the TSRP and the TSNRP were discussed in the qualitative section of the research. Each prospective teacher was interviewed for 20 minutes. These interviews were recorded with a voice recorder. In the interviews, the prospective teachers were requested to express what they thought of the questions that were given in the TSRP and the TSNRP Furthermore; their opinions were taken about the questions that they answered incorrectly. The reason of requesting to express their ideas is to reveal whether they are aware of the mistakes they made in the process of problem solving and their ideas about the reasons of making these mistakes.

Data Analyses

The quantitative data of the research was collected with the measurement tool. This data was checked and graded. The information, which was obtained from this grading, was processed into the information forms and transferred to computer, and statistical analyses were conducted.

Regarding the analysis of the qualitative data of the research, the data obtained from the interviews, which were conducted with the prospective teachers. This was analyzed in the scope of the qualitative research approach. The descriptive analysis technique was used in analyzing the data obtained in the qualitative research. The data obtained in the descriptive analysis is summarized and interpreted in accordance with predetermined categories. In this analysis, direct excerpts are frequently featured in order to reflect the opinions of the interviewed or observed individuals in a striking way (Yıldırım & Şimşek, 2006).

In order to establish the reliability of the research, the answers given by the prospective teachers to open-ended questions were examined by the researcher and three field experts, and the items in which there was "Agreement" and "Disagreement" were identified. The formula stated below was used for the reliability of the research (Miles & Huberman, 1994). In order to conclude that the study is reliable from the coding reliability study of the researchers, a minimum 70% reliability value must be reached (Yıldırım & Şimşek, 2006). An 80% value was found in this calculation, and the research was considered reliable.

Reliability= [(Agreement)/(Agreement)+(Disagreement)]×100

Results

The findings, which were obtained from the data collected via measurement tools throughout the research, are featured in this section. General statistical information for the TSRP and the TSNRP were given at first. Then, the findings obtained from all data were arranged and presented in accordance with the order of the sub-problems of research.

General Findings on the TSRP

A success test named the TSRP, which was prepared by the researchers, was presented to the participants in this research. The answers given to the questions that required performing operations by the prospective teachers using their trigonometry knowledge was examined. These prospective teachers were first-year students at the Department of Elementary Mathematics Instruction at the Faculty of Education. There are five questions in this success test. Distribution of the prospective teachers, who answered the TSRP both correctly and incorrectly, is given in Table 2.

Table 2: Distribution of the answers given by the prospective teachers to the TSRP.

Trigonometry	Correct		Incorrect	
	f	%	f	%
TSRP1	51	96.2	2	3.8
TSRP2	51	96.2	2	3.8
TSRP3	50	94.3	3	5.7
TSRP4	52	98.1	1	1.9
TSRP5	39	73.6	14	26.4

When Table 2 is examined, it is observed that the prospective teachers are generally successful in the questions that require trigonometry knowledge. In particular, we can state that the ratio of giving correct answers is 95% except for Question 5. It was found that 26.4% of the prospective teachers gave wrong answers to Question 5. "Cosine theorem" must be used to solve this question. The reason for the incorrect answers by the prospective teachers might be the fact that this theorem requires memorization and it is used less. In the light of this information, we can state that the prospective teachers were successful, especially in answering procedural knowledge questions that required trigonometry knowledge.

General Findings on the TSNRP

A success test named TSNRP, which was prepared by the researchers, was presented to the participants. This research examined the answers given to the questions that required reasoning by the prospective teachers using their trigonometry knowledge. These prospective teachers were first-year students at the Department of Elementary Mathematics Instruction at the Faculty of Education. There are five non-routine questions in this success test. Distribution of the prospective teachers who answered the TSNRP correctly and incorrectly is seen in Table 3.

Table 3: Distribution of the answers given by the prospective teachers to the TSNRP

Trigonometry	Correct		Incorrect	
	f	%	f	%
TSNRP1	17	32.1	36	67.9
TSNRP2	43	81.1	10	18.9
TSNRP3	41	77.4	12	22.6
TSNRP4	3	5.7	50	94.3
TSNRP5	33	62.3	20	37.7

When Table 3 is examined, it is observed that the ratio of those who gave a "correct" answer to Problem 1 among the other problems that were featured in the TSNRP to the total number of prospective teachers is 32.1% whereas the ratio of those who gave a "correct" answer to Problem 4 to the total number of prospective teachers is 5.7%.Regarding problems 2, 3 and 5, it was found that the ratios of those who gave a "correct" answer to the total number of prospective teachers were 81.1%, 77.4% and 62.3% respectively.

It is noteworthy that the prospective teachers had particular trouble with Question 1 and Question 4. The questions in the TSRP and the TSNRP, which were used as data collection tools, were the same in numerical terms. However, these questions were asked in the TSRP as routine and procedures whereas in the TSNRP as non-routine and verbal problems. As seen from Table 2, it is observed that more than 95% of the prospective teachers gave a correct answer to both Question 1 and Question 4. This can be stated that the prospective teachers were more successful in routine and procedural questions, but they were unsuccessful in non-routine questions containing the same numbers that required transferring trigonometry knowledge into the problem situation. The reason for this failure can be associated with the fact that the prospective teachers have good procedural knowledge and they generally solve procedural questions in trigonometry subjects in the courses.

Difference between Success in the TSRP and Success in the TSNRP

The descriptive statistics of the prospective teachers' scores in the TSRP and the TSNRP scores are given in Table 4 in order to answer Sub-problem 1 of the study, which was "Is there a significant difference between the prospective teachers' success in the TSRP and success in the TSNRP?".

Table 4. Descriptive statistics of the prospective teachers' scores in the TSRP and the TSNRP

	n	$\overline{X}$	s	t
TSRP	53	4.59	.63	-17.032*
TSNRP	53	2.59	.95	

*p<.05

When Table 4 is examined, it is observed that there is a difference between the prospective teachers' score average in the TSRP ($\overline{X}$= 4.59) and their score average in the TSNRP ($\overline{X}$= 2.59). In view of the t-test for relational samples conducted in order to determine whether this difference was statistically significant, it was found that the difference was significant (t(52)=-17.032, p<.05).

Relationship between the TSRP Questions and the TSNRP Questions

Correlation of the prospective teachers' scores on each of the TSRP and the TSNRP questions was examined in order to answer Sub-problem 2 of the study, which was "Is there a significant difference between the prospective teachers' TSRP questions and TSNRP questions?" These values are given in Table 5.

Table 5. Relationship between the Scores in the TSRP Questions and the TSNRP Questions

	TSRP1	TSRP2	TSRP3	TSRP4	TSRP5
TSNRP1	.136 .331				
TSNRP2		.158 .260			
TSNRP3			.063 .656		
TSNRP4				.034 .809	
TSNRP5					.770 .000*

When Table 5 is examined, it is observed that there is a high positive relationship between the scores in the TSRP5 and the TSNRP5 that represented the same numerical situations (r=.770, p<.01). No significant relationship was found among the other questions.

How do the prospective teachers explain their mistakes in the TSRP?

When Table 2 is examined, it is observed that the prospective teachers had trouble in answering Question 5 in the TSRP. It was found that 26.7% of the prospective teachers (14 prospective teachers) gave incorrect answers to this question. This question is given in Figure 3.

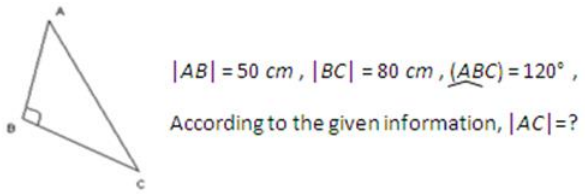


Figure 3: Question 5 in the TSRP

In the interviews conducted with the prospective teachers regarding the reasons for giving an incorrect answer to this question, the prospective teachers stated that they tried to use "cosine theorem" in solving the question. Some of the prospective teachers stated that they had trouble remembering "cosine theorem" that is a memorized knowledge (4 of 14 prospective teachers). The answers from several prospective teachers, who used this theorem incorrectly, are given in Figure 4.

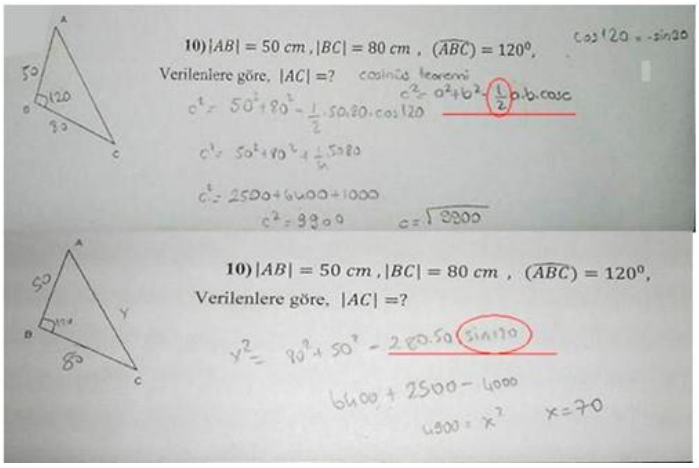


Figure 4: Answers given to Question TSRP5 by M5* and F4* among the prospective teachers

** M5: Number 5 male prospective teachers, F4: Number 4 female prospective teacher. (all excerpts will be given with this coding system.)*

10 out of 14 prospective teachers, who gave incorrect answers, stated that their incorrect answers resulted from procedural mistakes. When the answers from the prospective teachers were examined, it was observed that they gave incorrect answers due to procedural mistakes, even though they correctly wrote "cosine theorem". These mistakes generally resulted from square root operation, the four basic operations and incorrectly writing "cos 120°". Examples of the answers given by these prospective teachers are given in Figure 5.

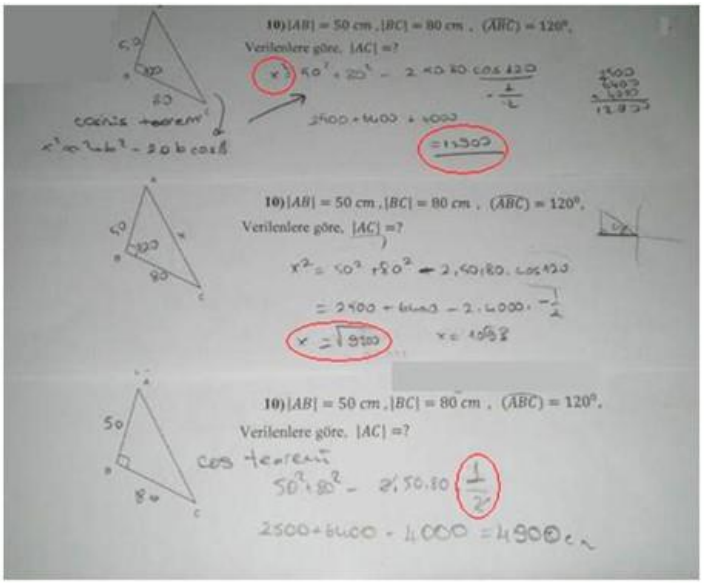


Figure 5: Answers given to Question TSRP5 by M24, F17 and F13 among the prospective teachers

When Figure 5 is examined, it is observed that prospective teacher 1 correctly found the number 12900, but he did not perform square root operation; prospective teacher 2 made an addition mistake and found 9300 instead of 12900; and prospective teacher 3 wrote "cos 120°" value as +1/2 instead of -1/2.

How do the prospective teachers explain their mistakes in the TSNRP?

When Table 3 is examined, it is observed that the number of prospective teachers who gave incorrect answers in TSNRP is considerably high. These prospective teachers were asked why they gave incorrect answers and later the answers of the prospective teachers were being recorded. After these recordings were examined and revealed, the prospective teachers organized their mistakes under four headings. The percentage and frequency information of these categories are given in Table 6.

Table 6: Frequencies and percentages of the categories related to the explanations of the prospective teachers for their mistakes

Categories (C)	TSNRP1		TSNRP2		TSNRP3		TSNRP4		TSNRP5		TOTAL	
	f	%	f	%	f	%	f	%	f	%	f	%
C1. Failure to Perform Modeling	18	50.00	1	10.00	4	33.33	25	50.00	4	20.00	52	40.63
C2. Conceptual-Memorization	4	11.11	1	10.00	1	8.33	4	8.00	0	0	10	7.81
C3. Failure to Understand the Question	4	11.11	3	30.00	2	16.67	7	14.00	1	5.00	17	13.28
C4. Procedural Mistake	0	0	1	10.00	3	25.00	3	6.00	9	45.00	16	12.5
No Comment-Out of Category	10	27.78	4	40.00	2	16.67	11	22.00	6	30.00	33	25.78
Total	36	100	10	100	12	100	50	100	20	100	128	100

According to Table 6, the categories that resulted from the explanations of the prospective teachers for their mistakes are as follows:

C1: Failure to Perform Modeling

C2: Procedural – Conceptual /Memorization

C3: Failure to Understand the Question

C4: Procedural Mistake

Findings about these categories are given under separate headings below.

Explanations of Those Who Gave Incorrect Answers Due to Failure to Perform Modeling

Olkun and Yeşildere (2010) defined mathematical modelling as representation of a real-life problem or situation with mathematical symbols or objects and stated that modelling can be achieved with concrete materials, drawings and symbols. Based on this opinion, it revealed that the prospective teachers said that they mostly had problems in visualizing and symbolizing the problems. Therefore, the category was named as "not making modelling".

The prospective teachers tried to transform these into mathematical language in order to solve verbal problems, but they were not successful. When Table 6 is examined, the problem of failure to perform modeling is observed in 52 incorrect answers (40.63%) in the interviews conducted regarding 128 incorrect answers given by 53 prospective teachers.

It is observed that the number of prospective teachers who stated that they gave incorrect answers to Question TSNRP 1 and Question TSNRP 4 due to failure to perform modeling constitute 50% of the total number of prospective teachers who gave incorrect answers to these questions (see Table 6). Some excerpts from the answers given by prospective teachers who were not able to perform modeling in Question TSNRP 1 are given below.

Female prospective teacher 1 (Here in after, female prospective teachers will be coded as F and male prospective teachers will be coded as M)

F1-TSNRP1: I was not able to represent the bridge and sea level as shapes. If it had been given with a shape, it would have been easier for me to solve the question as it would have seemed more visual to me and I would have understood the question.

F20-TSNRP1: I made a mistake while I was representing the question as a shape.

M41-TSNRP1: I was not able to reach the solution since I had trouble in representing the verbal expressions in the question as shapes. I experienced difficulty in configuring the concepts used in the question in my mind.

This can also be understood in the excerpts from the prospective teachers, stating that they were generally familiar with figure questions. They initially tried to represent the question as a shape in the non-routine problems that were not given as shapes, as in TSNRP, and that were in the form of verbal problems. However, it was found that they were not able to represent the question as a shape since they were not able to perform modeling.

F3-TSNRP4: I struggled to draw the shape, which emerged in my head when I read Question 4, on the paper. We have trouble in representing the problem as a shape since the instruction we receive is oriented towards solving readily available things and does not allow us to make interpretations.

M43-TSNRP4: I did not try to write the question on the paper. I directly applied the solution to the question that I built in my mind.

Many of the prospective teachers stated that they gave incorrect answers because they were not able to represent the problem situation as a shape or a numerical expression whereas M43 stated that he visualized the shape of the question in his mind and tried to find a solution. However, this also caused erroneous modeling.

Explanations of Those Who Gave Incorrect Answers Due to Conceptual Knowledge and Memorization

Some prospective teachers stated that they did not have trouble in procedural sections while solving the questions. They reached the solution by performing the procedure based on memorization, especially when the question was given as a shape. They failed in the TSNRP because they did not have a solid grasp of the subject. When Table 6 was examined, it was observed that 10 of the prospective teachers (7.81%) gave incorrect answers in the TSNRP due to the use of procedural knowledge and memorization.

The prospective teachers stated the reason for their incorrect answers to question TSNRP 1 and question TSNRP 4 to perform modeling was particularly the use of procedural knowledge and memorization (four prospective teachers, see Table 6.). Some excerpts from the prospective teachers who stated that they had made mistakes in TSNRP 1 and TSNRP 4 due to procedural knowledge and memorization are given below.

M30-TSNRP4: The reason for my incorrect answer is the fact that I misunderstood the question conceptually, but I was able to solve it procedurally.

F20-TSNRP1: While I am performing the procedures or transforming the given verbal expressions into shapes, I do what I am used to doing. I succeed in the same question styles, but when the same question is asked in a conceptually and verbally different way, I make mistakes since I reach the solution by means of memorization.

In view of the answers given by the prospective teachers, it was found that the questions were not understood conceptually. Furthermore, the prospective teachers tried to implement certain solution methods that they had experienced before – as is – while performing the procedures.

Some prospective teachers associated conceptual knowledge and memorization with our education system. The prospective teachers stated that it was difficult to change the system, although they were aware of this situation. They also stated that they tried to take notes and pass the class instead of understanding and learning. A prospective teacher's answer, which is relevant to this explanation, is given below.

F46: I am a person of procedures. I do not understand when I am asked to multiply 3 by 5 indirectly. However, everything becomes very simple when it is given as "3×5=?". In my opinion, the problem not only lies with the student. We have always been given procedures so far. The system directs the students to memorization and procedures. There is no longer a generation of teachers who want to learn, understand and improve themselves. What we have are teachers who try to take notes, pass the class and earn money as soon as possible. We are not willing to learn; but to memorize instead... We are the generation of procedures and memorization.

Explanations of Those Who Gave Incorrect Answers Due to Failure to Understand What Was Read

When the answers of the prospective teachers were examined, it was observed that they gave incorrect answers to some questions due to failure to understand what they read. For 17 of 128 incorrected answers made in the test, it was found that some of the prospective teachers stated that they did not understand "what was given" and "what was asked" in the question when they read it. These prospective teachers attributed their failure to understand the problems with the fact that they did not generally read books.

M31: Sometimes, I have trouble in understanding or visualizing what I read in normal life. I attribute the reason for this to the fact that I am not a person who reads books much.

M35: Students understand one way or another when the questions are kept short. However, they begin to fail to understand as the question gets longer. In my opinion, the only reason for this is the fact that we do not read books much.

Furthermore, there are also prospective teachers who were not able to visualize the concepts in their minds since they did not understand what was given in the question. These prospective teachers stated that they did not come across such question types before.

M35: The reason is the fact that I do not understand the words much. I do not make mistakes in procedures.

M30-TSNRP1: The reason for my mistake is the fact that I have difficulty in understanding.

F27: I did not understand what was signified in the question. Since we are generally given shapes, I got confused a little. We do not generally perform procedures from abstract things. I was not able to understand when it was written down.

Explanations of Those Who Gave Incorrect Answers Due to Procedural Mistake

The prospective teachers attributed some of their mistakes in the TSNRP to the fact that they made procedural mistakes. The prospective teachers could model, make sense of the problems while answering and know what to apply conceptually, however, they made mistake in the solution process. It was found that 16 of 128 mistakes stemmed from procedural mistakes. The prospective teachers stated that they generally make mistakes in four basic operations and remembering, making operations quickly with square root and trigonometric theorems. It was observed that they made this mistake particularly in Question TSNRP5. The answers of some prospective teachers who made this mistake are given below.

F8-TSNRP5: I mistook the angle in the question for something else. Thus, I implemented operations and cosine theorem incorrectly.

F37-TSNRP5: I took cos 120° as (1/2) instead of (-1/2). That is why I made a mistake. It is because of my lack of knowledge. That is why I acted carelessly while wanting to perform it quickly.

F39-TSNRP5: I made a mistake because I wrote sine in cosine theorem. It resulted from the fact that I forgot it.

Some of the prospective teachers stated that they drew the shape differently in order for the answer to be a whole number, especially inTSNRP2. This indicates that the prospective teachers thought it was always necessary for the answer to be a whole number while solving problems.

M10-TSNRP2: I generally followed a way based on memorization because I thought that the result would be a whole number. I modified the question to make an angle of thirty degrees with the wrong side in order to find a whole number as the answer.

Discussion, Conclusion and Suggestions

Results related to the findings will be discussed under two main headings as quantitative and qualitative in this section. Suggestions will be made in view of these results.

Results Obtained from the Quantitative Findings

In view of the findings obtained from the research, it was observed that the ratio of giving correct answers to the questions in the TSRP that required procedural knowledge by the prospective teachers was high. Thus, it can be stated that the prospective teachers did not have trouble in the questions that required procedural trigonometry knowledge. In their study, Soylu and Soylu (2006) also stated that the students did not experience difficulty in the exercises that required, in particular, procedural knowledge. In their study, Sam, Lourdusamy and Ghazali (2001) stated that the students were more successful in solving procedural questions than solving the verbal problems. On the other hand, it was found that prospective teachers made quite a lot of mistakes in the TSNRP questions that required transferring their trigonometry knowledge to verbal problem situations. In such non-routine questions, the prospective teachers are expected to transform the question given in the form of a verbal problem into mathematical language using their mathematical knowledge. It was found that the prospective teachers who participated in the study fell short in transforming the questions into mathematical language and numerical expressions. Research conducted shows that transforming verbal problem situations into equations or numerical expressions is one of the issues that students from different grades experience difficulty within the problem solving process (Akkan, Baki & Çakıroğlu, 2012).

Mathematics educators divide the mathematical knowledge into two as procedural and conceptual. Procedural knowledge signifies the mathematical knowledge on rules and operations whereas it is

important to infer in conceptual knowledge. However, it is stated that these two types of knowledge constitute an inseparable whole in terms of mathematics instruction (Olkun & Toluk-Uçar, 2012, p. 28-29). When we examine the prospective teachers who participated in the study, we can state that their procedural trigonometry knowledge is considerably high. However, it is observed that they do not have a solid grasp of conceptual knowledge in which the meaning is important and which is required while transforming the given problem situation into mathematical language. In their study, Arslan and Altun (2007) stated that when the students come across a problem, they tend to take a look at the problem and reach the result by quickly implementing the necessary operations on the given numbers. The results of the study support the results of the research of Arslan and Altun (2007). According to Sabella and Redish (1995), the really difficult thing for students is not learning algorithmic calculations but mathematical concepts. This result corresponds to the findings obtained in our research.

On the other hand, it is observed that there is a difference between prospective teachers' success rates in the TSRP and the TSNRP. The findings of the study revealed that this difference was statistically significant. It was set forth in the conducted studies that there are differences between prospective teachers' success in the tests on routine problems and the tests on non-routine problems (Işık & Kar, 2011; Aladağ, 2009). In their study, Akgün et al. (2012) state that students are considerably successful in the questions where they use procedural knowledge, but they are less successful in the questions that involve, in particular, verbal problems that require transferring and using this knowledge.

When the TSRP and the TSNRP are examined, it is possible to state that the questions in these two tests are the same in numerical terms. However, the prospective teachers were more successful in the TSRP. For this reason, the TSRP and the TSNRP were compared question by question, and no relationship was found among Questions 1, 2, 3 and 4.This can be argued that the reason was the fact that they gave correct answers to the questions in the TSRP whereas they gave many incorrect answers to the questions in the TSNRP where they were expected to transfer their trigonometry knowledge. A high positive relationship was found between TSRP5 and TSNRP5. The necessity of using cosine theorem, which is rather based on memorization and is less used knowledge, in answering this question might have lead to the higher number of incorrect answers from prospective teachers for TSRP5.

Results Obtained from the Qualitative Findings

When the answers given to TSRP by the prospective teachers are examined, it is observed that there were no mistakes except for Question 5. It was found that 14 prospective teachers gave incorrect answers to Question 5. When these prospective teachers were asked about the reasons for their mistakes, they stated that they generally gave incorrect answers due to procedural mistakes and incorrect use of cosine theorem. Furthermore, the prospective teachers stated that they had particular trouble with using cosine theorem. This knowledge is not used much in the courses. Instead, Pythagorean Theorem, which is a special form of this theorem, is used more in the courses. The prospective teachers stated that they gave incorrect answers since they were not able to remember cosine theorem. Battista (1999) stated that less used mathematical concepts are forgotten sooner.

When the reasons given by the prospective teachers for the incorrect answers to the TSNRP were examined, it was found that these reasons could be discussed under four different categories. The first of these categories can be taken as the failure to transform the given problem situation into a shape and a numerical expression. This category was named as "failure to perform modeling" by the researchers. The prospective teachers experienced difficulty in transforming the question into a shape when they read the question. Therefore, they were not able to write it as a numerical expression. In the conducted studies, it is stated that the courses given via mathematical modeling increase the success of prospective teachers (Sağırlı, Kırmacı, & Bulut, 2010).

The second category is the category of "failure to understand what is read". Polya (1957) suggested four stages while solving a verbal problem, and stated that the first of these stages is to understand the problem. The prospective teachers who participated in the study stated that they did not find the solution or they found an incorrect solution since they were not able to understand the problem. In the study conducted by Güner and Alkan (2011), it was found that the students who were able to understand what was being asked in the question were able to solve that problem easily whereas the great majority of the students were not

able to answer nearly half of the questions that were asked in the research since they were not able to understand these questions.

Some of the prospective teachers stated that they gave incorrect answers to TSNRP due to the use of procedural knowledge and memorization. This expression was taken as the third category. The prospective teachers stated that they always performed procedures in their courses and they tried to memorize formulas and rules and use this memorized knowledge in the questions. Ersoy (2002) argued that procedural knowledge could be learned by memorization whereas one must understand in order to learn conceptual knowledge (Ersoy, 2002). Although one of the aims of mathematics instruction is the skill of performing procedures, it is considered important for the prospective teachers to perform procedures by knowing the meaning of the learned mathematical concepts and where it must be used (Olkun & Toluk-Uçar, 2012; Altun, 2005). A few prospective teachers stated that they gave incorrect answers since they made mistakes while using their procedural knowledge and they also forgot the rules.

Suggestions

The fact that the concept of trigonometry is frequently discussed from its algorithmic (procedural) aspects can cause its conceptual dimension to be neglected. Therefore, field education experts must give information about the conceptual dimension of the concept of trigonometry apart from its procedural dimension to prospective teachers in courses. Furthermore, questions in the form of verbal problems must also be featured while solving questions about trigonometry in the courses.

Verbal problems can be prepared for not only the subject of trigonometry but also all mathematical concepts, and prospective teachers can be requested to read these problems and explain what they understand in the problems. Thus, prospective teachers may become more adequate in understanding what they read, which constitutes the first problem solving step. The second step, which is required from a prospective teacher who understands the problem, is to plan and transform the given problem into mathematical language or a numerical expression. In order to achieve this, they have to visualize what is given in their minds and transform it into a shape or perform mathematical modeling. Therefore, mathematical modeling activities must be performed in the courses.

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