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The Relationship among Pre-Service Mathematics Teachers' Conceptual Knowledge, Opinions Regarding Proof and Proof Skills

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The aim of this study is to reveal the relationship among pre-service mathematics teachers' conceptual knowledge, opinions regarding proof and proof skills in terms of the definition of the convergence of a sequence. The participants of the research are composed of six pre-service mathematics teachers who are third-year students at the department of elementary mathematics teaching in a state university located in Turkey. The data of the research were collected via semi-structured interviews. According to the findings of the research, it was found that the majority of the pre-service teachers who participated in the study did not have a sufficient level of conceptual knowledge on the definition of convergence of sequence; that their opinions on proof were negative; and that their level of success in proving the theorem, which they had proven before, was considerably low. Furthermore, it was found in the research that there was a strong relationship among pre-service teachers' conceptual knowledge, opinions regarding proof and proof skills, and the academic success achieved in the related course did not reflect on their proof skills.

Introduction

One of the most important aims of mathematics instruction is to ensure the development of obtaining logical answers to "why" and "what for" questions, that is to say, the development of reasoning (Altıparmak & Öziş, 2005). For the mathematics instruction to achieve this aim, mathematical proofs constitute an important tool in terms of dealing with not only the correctness of a statement but also why it is correct. Mathematical proofs are regarded as the most important factor in learning mathematics because they involve associating judgments in a certain mold and being sure of these judgments in view of the development of mathematics and realities or propositions (Knuth, 2002).

When mathematical proofs are evaluated in terms of mathematics instruction, they provide a considerable amount of benefits to the students in mathematical terms. The development and maturation of students' mathematical knowledge through mathematical proofs (Kitcher, 1984), developing the skill of critical thinking in students and offering new methods with which students can solve problems (Rav, 1999) constitute just some of these benefits.

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Although mathematical proofs are regarded as an important part of mathematics instruction (Güven, Çelik & Karataş, 2005), the building block of mathematics (Heinze & Reiss, 2003) and the objective of advanced mathematics (Weber, 2001) by many researchers, proof is a concept about which students from every level experience difficulty and have prejudice (Almeida, 2003; Arslan, 2007; Arslan & Yıldız 2010; Coşkun, 2009; De Villiers 1990; Jones, 2000; Moore, 1994; Raman, 2003). When the difficulties experienced by the students during proof processes are examined, we see that Moore (1994) listed these difficulties as failure to state the definitions; failure to understand the meanings of the concepts intuitively; failure to use concept images during proof process; generalization and insufficiency in using examples; failure to know what kind of a proof structure to form in view of the definitions; failure to understand mathematical language and notations; and lastly, failure to know how to begin the proof process. One of the most common difficulties experienced in performing proof is the lack of conceptual knowledge regarding the concepts with which the proof is related. This difficulty experienced by the students is a common finding that was obtained in the researchers conducted by different researchers (Gibson, 1998; Knapp, 2005; Moore, 1990; 1994; Weber, 2006;). Another factor affecting students' mathematical proof processes is their opinions regarding proof. Beliefs and opinions are not only related to attitudes, but also directly affect proof processes. This is because beliefs are influential in selecting proof method and strategy, which stands as a difficulty in proof processes (Furinghetti & Morselli, 2009). In this regard, it can be stated that opinions regarding proof affect the success of activities involving proof.

When the reasons for the difficulties experienced by the pre-service teachers in proof process are examined, it is observed that the mathematics courses given in universities are filled with proofs that are difficult to understand for pre-service teachers at first glance, and consequently, pre-service teachers memorize proofs without understanding them in order to pass these courses (Conradie & Frith, 2000). Since these courses in universities are generally given via the traditional method, students mostly take the exams without internalizing the concepts and instead memorizing the theorems and contents of the proofs without even knowing them, and they pass the exams. Furthermore, according to mathematicians, mathematical proof is filled with key ideas rather than the information existent in it. However, mathematicians do not place the necessary emphasis on the key ideas during instruction, and more importantly, they do not use these ideas during evaluation (Raman, 2003). Formation of valid ideas and proofs along with criticism of the ideas is an integral part of performing mathematics. If this reasoning skill is not earned by students, mathematics becomes a process of following an operation sequence and imitation of examples without considering what operation sequence means (Ross, 1998).

When the difficulties experienced by the pre-service teachers are taken into account, we can generally talk about the existence of two conditions. One of these conditions is that the pre-service teachers do not have a correct idea about mathematical proof and they cannot even remember the proof (Chazan, 1993; Moore, 1994; Weber, 2001), whereas the other condition is that they do not understand the concepts and theorems and implement them in a systematized manner. The fact that the pre-service teachers remember a theorem or a concept does not guarantee that they will perform the proof. They fail in performing the proof and come to an impasse as they do not know what to do. In order to understand the reasons for this failure, it is more useful to understand the methods utilized by the pre-service teachers while they try to perform proof (Weber, 2001). The aim of this study is to reveal the relationship among pre-service mathematics teachers' conceptual knowledge, opinions regarding proof and proof skills in terms of the definition of convergence of sequence. Answers were sought to the following research problems within the scope of this aim.

1. What is the level of pre-service teachers' conceptual knowledge regarding the subject of convergence of sequence?
2. What are the pre-service teachers' opinions regarding proof?
3. What is the level of pre-service teachers' proof skills on the subject of convergence of sequence?

Method

Taking the qualitative research approach as a basis, the case study model was used in the study. It was concluded that the most appropriate case study design for this study was the holistic multiple-case study. This is because there is more than one case that can be perceived as holistic alone in this design. Each case is studied holistically in itself and later compared with each other (Yıldırım & Şimşek, 2006). In this study, three cases were examined, namely pre-service mathematics teachers' conceptual knowledge, opinions regarding proof and proof skills in terms of convergence of sequence. Then, these sections were studied holistically and compared with each other.

Research Group

This study was conducted with six pre-service mathematics teachers who are studying at the department of elementary mathematics teaching in a state university located in Turkey. The pre-service teachers are third-year students at the department of elementary mathematics teaching, which is a four-year department. The average age of pre-service teachers (three of whom are females and the remaining three are males) is twenty two. The names used in the research are not the real names of the pre-service teachers. The names used are the aliases designated by the researchers. While selecting the participants, necessary attention was paid to the fact that their knowledge on the subject of sequence is easy to remember. For this purpose, the research was conducted directly after the Analysis 3 course in which the subject of sequence was given. While selecting the participants, students who successfully passed the Analysis 3 course and who had different letter grades were preferred. Therefore, maximum variation sampling, which is among the purposeful sampling methods, was used in selecting the participants. This is because the aim in a sampling, which is based on maximum variation, is not for generalizations. On the contrary, the aim is to attempt to determine whether or not there are common or shared phenomena among the cases that exhibit variation, and to set forth the difference aspects of the problem according to this variation (Yıldırım & Şimşek, 2006). In the university where the study was conducted, students who pass the course are given letter grades, namely AA, BA, BB, CB, CC, DC and DD ranging from the most successful to the least successful. Since there were no students who passed the Analysis 3 course with the letter grades DC and DD, these students were not included in the research group. Instead, one more students with the letter grade BA were included in the research group, considering that he/she might be able to provide more information related to the subject. While selecting the research group, students who successfully passed Analysis 3 course were listed according to their letter grades. Then, participants were randomly selected from these lists. The aliases and letter grades of the participating pre-service teachers are as follows: Aydın (AA), Burak (BB), Cemil (CC), Bade (BA), Banu (BA), and Ceren (CB). In view of the letter grades achieved in the Analysis 3 course, Aydın was the most successful student whereas Cemil was the least successful student.

Data Collection

The data of the research were obtained via semi-structured "Proof Process Interview Form" (PPIF). The PPIF is composed of three sections. In the first section, the participants were given the definitions of sequence and convergence of sequence that were required for proving the theorem in the third section. The aim of this section is to observe the degree to which the participants understood and internalized these concepts and see whether or not they could perform the proof when they had the necessary definitions. In the second section of the PPIF, there are open-ended questions that were asked in order to reveal the pre-service teachers' opinions regarding proof. In the third section, in order to reveal the proof skills of the participants, they were requested to prove the "$(s_n) \longrightarrow s$ and $(t_n) \longrightarrow t \Rightarrow (s_n + t_n) \longrightarrow s + t$" theorem again. While preparing the PPIF, the opinions of three academicians – two of whom were lecturers specializing in qualitative research methods and one of whom was a lecturer specializing in the analysis and theory of functions – were taken. Before the interviews conducted with the pre-service teachers, the researchers explained that the research would completely be conducted upon the principle of voluntariness, and the pre-service teachers who did not want to continue this study could leave the study whenever they wished. Furthermore, the pre-service teachers were told that the interviews would be recorded with an audio recorder. They were asked whether this condition would constitute an inconvenience for them. Their permissions were taken on this matter. The researchers stated that the names of and information about the pre-service teachers would not be shared with anybody, and aliases would be used instead of their real names in the research. The interviews conducted with the pre-service teachers lasted for 25-30 minutes. All interviews were conducted in an environment where the second author and the pre-service teachers could talk one-to-one and where it was believed that external factors would not distract the participants. The definitions and theorems used in the PPIF are given below.

Definition 1 (Sequence of Real Numbers): The function, which is defined as $f: N \rightarrow R$, is called sequence of real numbers.

Definition 2 (Convergence of Sequence): $(s_n) \longrightarrow s \Leftrightarrow$ for $\forall \varepsilon > 0$, $\exists n_0 \in N$ such that for $\forall n > n_0$, $|s_n - s| < \varepsilon$.

Theorem: $(s_n) \longrightarrow s$ and $(t_n) \longrightarrow t \Rightarrow (s_n + t_n) \longrightarrow s + t$.

Analysis of the data

Descriptive analysis was used in analyzing the data obtained from the opinions of pre-service teachers. The data obtained in descriptive analysis are summarized and interpreted according to the themes that have been designated before (Yıldırım & Şimşek, 2006). Data triangulation was used in the study. The participants were requested to think aloud. Written statements and audio records were evaluated together. Firstly, audio records were transcribed in the study. While transcribing the data, discussions were made with the participants about the incomprehensible statements. Thus, incomprehensible statements were clarified. After the process of transcribing the interview data, the researchers classified the raw data according to the categories designated beforehand. Following the classification stage, the categories were examined by two colleagues of the researchers. Lastly, the feedback and categories obtained were reorganized. An attempt was made to present the written and verbal statements of the participants and their dialogues with the researcher in a frequently descriptive manner. Therefore, it was aimed to increase the reliability of the research data.

Findings

Findings regarding pre-service mathematics teachers' conceptual knowledge on the definition of convergence of sequence, their opinions regarding proof and proof skills are presented in this section.

Findings Regarding Pre-Service Mathematics Teachers' Conceptual Knowledge on the Definition of Convergence of Sequence

In the first section of the study, the participants were given the definitions sequence of real numbers and convergence of sequence that they would use while they were performing proof. The participants were requested to read aloud these definitions. The following questions were put to the participants in order to understand the degree to which they internalized the definition of convergence of sequence: "What do you understand from the convergence of a sequence towards a point?" and "What is the function of n_0 in the definition of convergence of sequence?" In view of the received answers, it was found that the participants' level of conceptual knowledge could be divided into adequate and inadequate categories. The categories of the participants are given in Table 1.

Table 1. Pre-service teachers' conceptual knowledge about definition of convergence of sequence

	Adequate	Inadequate
Bade	✓	
Banu		✓
Aydın		✓
Cemil		✓
Burak		✓
Ceren		✓

In Table 1, it is observed that among the participants, only Bade was able to explain the definition of convergence of sequence in her own words. That is to say, she internalized the concept of convergence of a sequence towards a point. In view of this, it is deduced that Bade's knowledge on the definition of convergence of sequence was at least at conceptual knowledge level. Bade's answers are given below.

Table 2. Bade's answers

What do you understand from the convergence of a sequence towards a point?	*What is the function of n_0 in the definition of convergence of sequence?*
Since the domain of this sequence is positive whole numbers, there will be more than one number. These numbers gradually approach towards a certain number. Here, a real number sequence. When sequence is a complex sequence, it can convergence towards a complex number, too. That is to say, it approaches a certain number within a certain rule. However, I cannot associate it with daily life.	*To my knowledge, n_0 is... There are many numbers in the sequence. Most of these numbers converge towards s. Several of these numbers do not converge. That is to say, they are finite. However, when approximately all of them converge, we consider them convergent. We do not regard them and we see them as mathematically small. In other words, we ignore them.*

It was found that Banu, Aydın, Cemil, Burak and Ceren were not able to explain the definition of convergence of sequence in their own words. That is to say, their knowledge on the definition of convergence of sequence was not at conceptual knowledge level. Banu and Burak's answers are given below.

Table 3. Banu and Burak's answers

What do you understand from the convergence of a sequence towards a point?	*What is the function of n_0 in the definition of convergence of*

		sequence?
Banu:	*Let me put it this way: in order for s_n sequence to converge towards a number, it is necessary that ε>0 and we have a n_0 natural number. The number, in which term of the sequence that we took converges towards negative, is a statement in the form of a small ε. I don't know how to express it verbally. However, I think it is symbolic.*	*Isn't this right? For instance, we take s_n and progress as s_1, s_2, s_3. We represent it with the natural number there. Can't we state it as that natural number? I thought we could. However, I don't know what exactly it is right now.*
Burak:	*We can call the value, which will be taken by a sequence in the limit condition for infinite cases, the place where that sequence converge. Can we say that the sequence exactly takes that value? Although it does not exactly take that value, it is near to it. As stated by this definition, we are talking about the values that it takes in the neighborhood of ε.*	*I cannot express it precisely.*

Findings Regarding Pre-Service Mathematics Teachers' Opinions on Proof

In the second section of the research, it was aimed to reveal the participants' opinions on proof from their statements. It was found that the participants' opinions on proof fell under "Positive" and "Negative" categories. The categories of the participants are given in Table 4.

Table 4. Pre-service teachers' opinions on proof

	Positive	Negative
Bade	✓	
Banu		✓
Aydın		✓
Cemil		✓
Burak		✓
Ceren		✓

In table 4, it is seen that only Bade had a positive opinion on proof. Bade's opinions on proof are given below.

Bade: I better understand the subjects on which I can perform proof. I believe so. That is because we take everything into account while performing proof. Therefore, when I perform the proof about linear independence in linear algebra and when I try to prove it on my own with pencil and paper, it becomes a better experience. Then, I don't forget what linear independence is. Similarly, there are limit definitions, convergence and related subjects in analysis, too. It becomes a more productive experience in them.

It was found that other participants had negative opinions on proof. Most of the participants regarded proof as an activity that must be memorized and that cannot be understood. The following statements of Ceren and Aydın, who are among the participants having negative opinions on proof, are as follows.

Ceren: Proof became a memorized subject in the way that it was given to us, or we were not able to think how this theorem could be proven. Can this theorem be proven with only this method? Is this the only solution to it? We never thought about it. That is to say, it was proven and we memorized it. It was like this, and similar rules can be applied to others. That is how we used it.

Aydın: We did not care about it much. Passing courses was enough for us. I think I have no interest other than passing courses... Theorems are long and hard to solve. It takes time to prove them. They are somewhat complex. I mean you accept something from different places. I feel that is wrong. It is right, but I forget it since it seems a little complicated. That is why we do not care about it much...

We rather tried to memorize the proof. Then, we performed it when they asked us about it. Other than that, we did not pay particular attention to it.

Findings Regarding Pre-Service Mathematics Teachers' Proof Skills About the Concept of Convergence of Sequence

After the participants' conceptual knowledge on the definition of convergence of sequence and their opinions regarding proof were examined, the participants were requested to prove the "$(s_n) \longrightarrow s$ and $(t_n) \longrightarrow t \Rightarrow (s_n + t_n) \longrightarrow s + t$" theorem, which they had proven before, in order to examine the pre-service teachers' proof processes. Before performing the proof, the participants stated that they had proven this theorem before and the definitions given in the definitions section were enough to prove this theorem. The conversation between Bade and the researcher about this subject is given below.

Researcher: In your opinion, what information do you require in order to perform the proof?

Bade: This definition is enough for me. I mean the definition of convergence.

...

Researcher: Have you ever seen such theorem before?

Bade: Yes, we have.

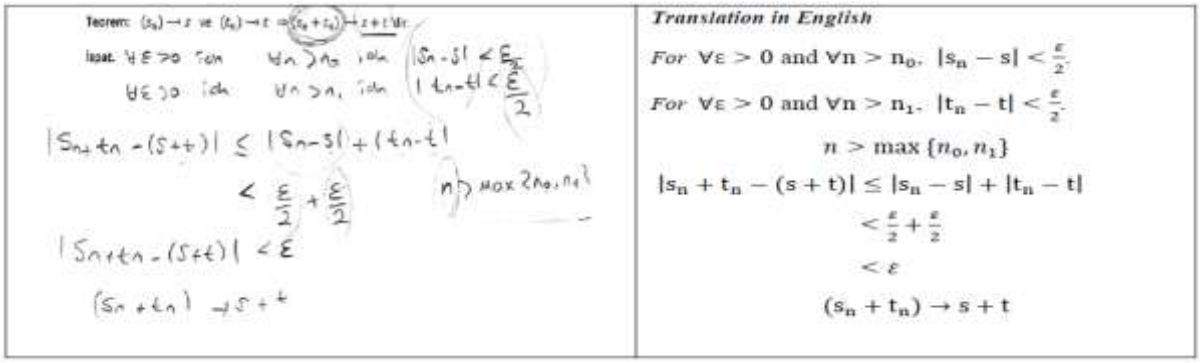
When the proofs performed by the participants were examined, it was found that the proofs came under four categories; namely, correct proof (the proof was written correctly and the explanation is sufficient), invalid proof (the proof was written correctly, but the explanation is insufficient), incorrect proof (the proof was written incorrectly) and incomplete proof (the proof wasn't written completely and it remained incomplete). The participants' states of performing proof are given in Table 5.

Table 5. Proofs performed by the pre-service teachers

	Correct proof	Invalid proof	Incorrect proof	Incomplete proof
Bade	✓			
Banu			✓	
Aydın		✓		
Cemil				✓
Burak			✓	
Ceren			✓	

When the performed proofs are examined, it is observed that Bade and Aydın correctly wrote the proof. However, Bade was able to explain the operations that she performed, that is to say, she was aware of what she wrote, whereas Aydın was not able to explain the operations that he performed. In the conducted interviews, it was observed that Aydın was not aware of the statements that he wrote since he memorized the proof of the theorem.

Figure 1 shows the proof performed by Bade and the conversation made by the researcher with Bade in order to understand whether or not she performed the proof consciously.



Translation in English

For $\forall \varepsilon > 0$ and $\forall n > n_0$. $|s_n - s| < \frac{\varepsilon}{2}$

For $\forall \varepsilon > 0$ and $\forall n > n_1$. $|t_n - t| < \frac{\varepsilon}{2}$

$$n > \max\{n_0, n_1\}$$

$$|s_n + t_n - (s + t)| \leq |s_n - s| + |t_n - t|$$

$$< \frac{\varepsilon}{2} + \frac{\varepsilon}{2}$$

$$< \varepsilon$$

$$(s_n + t_n) \to s + t$$

Figure 1. Proof performed by Bade

Researcher: What is n_0 breakpoint of $(s_n + t_n)$ sequence if it has any?

Bade: There is n_0. I wrote it here, too. We say every n>n_0. Our n will be bigger than both n_0 and n_1. Therefore, we chose the maximum one between n_0 and n_1.

Researcher: Why did you choose the maximum one?

Bade: Because it must be bigger than even the biggest one in order for it to include it. For instance, let us leave out the two terms before n_0 for s_n sequence. Let us ignore it. Let us leave out the five terms before n_1 for t_n sequence. If I had chosen the minimum one, if I had ignored two of them, that would not have happened. I mean it would not have included the others. In order not to make a mistake, I choose the maximum one for it to include all of them. I choose the maximum one between n_0 and n_1 for it to include the largest set.

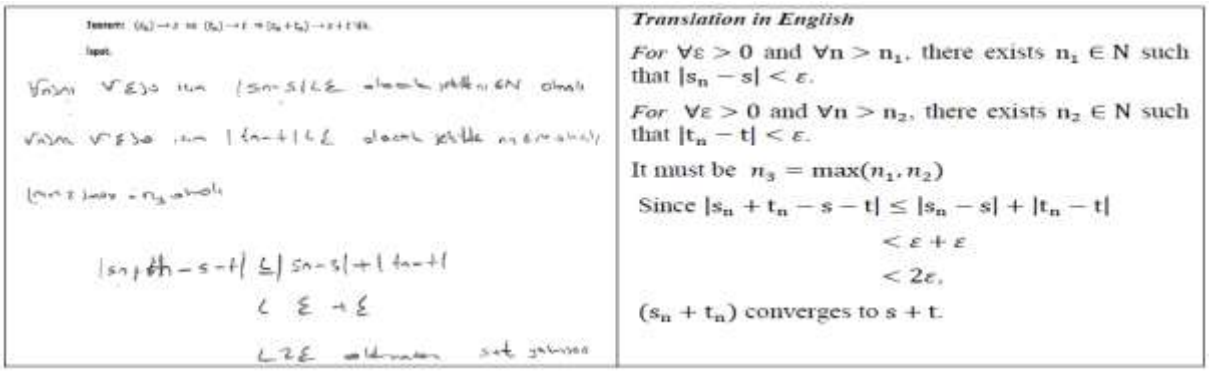
Examining Bade's statements, it was observed that she correctly proved the theorem by being aware of the statements that she wrote while proving the theorem. Bade developed a special systematic method while proving the theorem. This method resembles problem-solving steps and it is as follows:



While proving this theorem, Bade understood the statement of the theorem (understanding), wrote verbal statements in the theorem mathematically (analyzing the symbols), used the definition of convergence of sequence (utilizing the definition), utilized mathematical operations (operation), check the proof that she performed and whether or not there was an insufficiency or error (check) respectively. While checking her proof, Bade realized that she did not use the breakpoint of the $(s_n + t_n)$ sequence which enabled her to collectively use the inequalities that she obtained from the first and the second definitions. She corrected her mistake. Accordingly, she concluded that the statements that she wrote were valid for the terms after the biggest breakpoint among breakpoints in the first and second definitions (correction). She checked her proof once again and completed her proof (result). Bade's statements about check and correction steps are given below.

Bade: Let me check it for a moment... We forgot about the operation that we had performed on the finite number. Here, n is bigger than n_1 [first line] whereas n is also bigger than n_0 in the other one [second line]. Therefore, n will be bigger than the biggest value of both n_0 and n_1. Since n_0 and n_1 will be finite, the terms of $s_n + t_n$ sequence converging towards s+t will be infinite. It will be bigger than the terms that are bigger than $n_0$which is included in s neighborhood as well as the terms that are bigger than n_1 which is included in t neighborhood. As a result, the infinite terms are within s+t neighborhood.

Figure 2 shows the proof performed by Aydın and excerpts from the conversation made by the researcher with Aydın in order to understand whether or not he performed the proof consciously.



Translation in English

For $\forall \varepsilon > 0$ and $\forall n > n_1$, there exists $n_1 \in N$ such that $|s_n - s| < \varepsilon$.

For $\forall \varepsilon > 0$ and $\forall n > n_2$, there exists $n_2 \in N$ such that $|t_n - t| < \varepsilon$.

It must be $n_3 = \max(n_1, n_2)$

Since $|s_n + t_n - s - t| \leq |s_n - s| + |t_n - t|$

$< \varepsilon + \varepsilon$

$< 2\varepsilon.$

$(s_n + t_n)$ converges to $s + t$.

Figure 2. Proof performed by Aydın

Researcher: What is your purpose in writing $n_3 = max\{n_1, n_2\}$ *here?*

Aydın: I do not know why I did that. I remembered it from the courses. It is not a property. What we have in the result are two different values: n_1 *and* n_2*. It is not clear which one we must take or which one is bigger. I think we put the bigger one into the operation in order to prove the theorem.*

Researcher: Why do you put the bigger one into the operation?

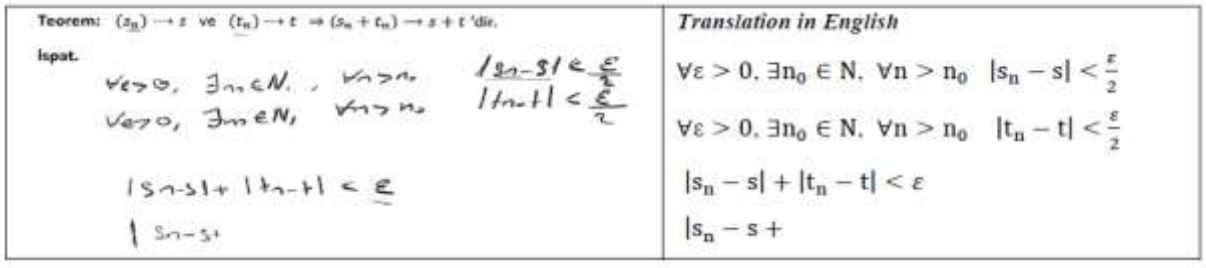
Aydın: We have never asked our teacher about it.

Researcher: Why did you write the statement with the maximum one in your proof?

Aydın: Memorization. Previous knowledge. It is memorization because I have seen it before.

Although Aydın correctly wrote the proof as figure, he performed the proof without being aware of what he wrote. In view of the conducted interview, it was found that Aydın's knowledge on this proof came from only memorization. While performing the proof, he exhibited a result-oriented approach and did not consider the transitions, connections and key ideas among the statements. He performed the proof without being aware of what he wrote.

Cemil was not able to complete his proof. While performing the proof, he tried to use the definition of convergence of sequence, other similar theorems that he knew and his memorized knowledge was the most striking feature. The proof performed by Cemil is given below.



Translation in English

$\forall \varepsilon > 0, \exists n_0 \in N, \forall n > n_0 \quad |s_n - s| < \frac{\varepsilon}{2}$

$\forall \varepsilon > 0, \exists n_0 \in N, \forall n > n_0 \quad |t_n - t| < \frac{\varepsilon}{2}$

$|s_n - s| + |t_n - t| < \varepsilon$

$|s_n - s +$

Figure 3. Proof performed by Cemil

It was found that Ceren, who was among other participants, only paid attention to the operational part of the proof while performing the proof. It was observed that she did not paid attention to the fact that in which cases that the inequalities she wrote during performing the proof were valid, and consequently, she did not write the condition that would allow for collectively using the two inequalities. Since the written inequalities would not be valid in this

case, the performed proof was evaluated as incorrect. In view of Ceren's proof, it is considered that she was not able to use the definition correctly since she did not comprehend the concept of convergence of sequence. The proof performed by Ceren is given in Figure 4.

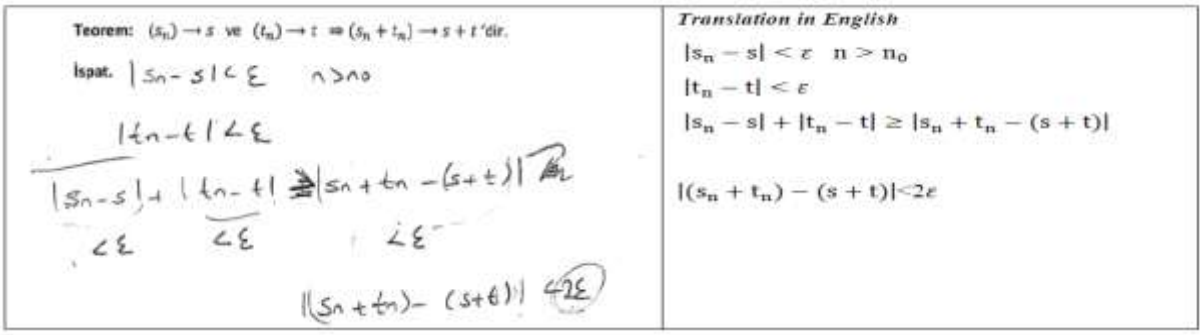


Figure 4. Proof performed by Ceren

While performing the proof, Burak and Banu correctly used the definition of convergence of sequence. However, like other participants, they did not question under which conditions the statements that they wrote were correct. If they had questioned under which conditions that they could perform this operation while combining the two inequalities, their proofs would have been correct. Since the proofs performed by them were invalid according to the definition of convergence of sequence, their proofs were evaluated as incorrect. This condition is observed in Burak's proof that is given in Figure 5.

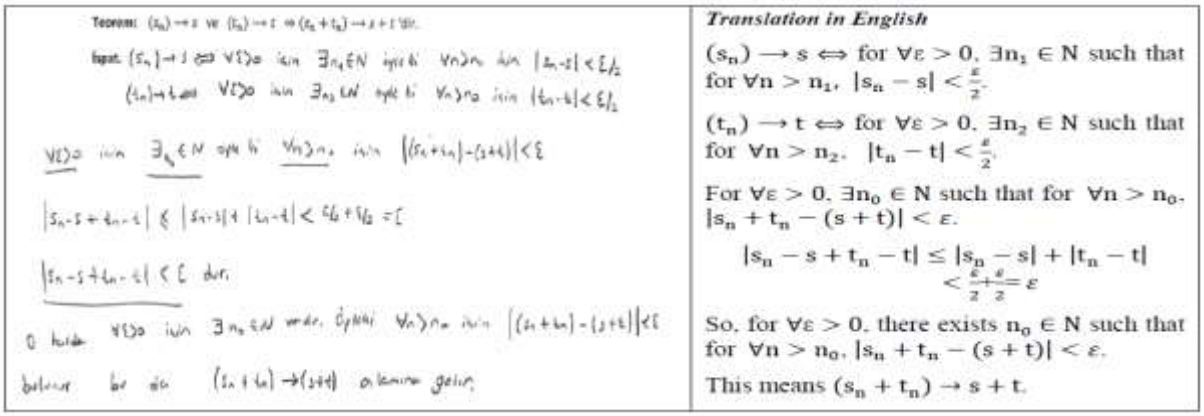


Figure 5. Proof performed by Burak

Results and Discussion

In regard to the first question of the research, questions were asked in order to reveal pre-service teachers' level of conceptual knowledge regarding the definition of convergence of sequence. At the end of the study, it was found that a great majority of the participants had a considerably low level of conceptual knowledge regarding the subject of convergence of sequence. It was found that only one pre-service teacher's (Bade) knowledge was at the conceptual knowledge level. The concept of convergence of sequence is a concept that is difficult for students to learn (Akbayır, 2004; Akgün & Duru, 2007). Furthermore, since the university-level mathematics courses are given with the classic lecture method as follows: definition→theorem→proof (Weber, 2004) or theorem→proof→examples (Almeida, 2003), many students cannot learn the concepts completely and form relations between the concepts (Yıldız, 2006). When looked at from this viewpoint, it can be considered that the knowledge of the students who participated in the study remained at knowledge level, and they were not able to understand the related concept. Accordingly, it can be suggested that the lecturers who

are responsible for giving the related courses must review their instructional methods and use different instructional methods.

The statements of the pre-service teachers were examined in order to find an answer to the second question of the research. Pre-service teachers' statements about their opinions on proof fell under positive and negative categories. It was found that the majority of the pre-service teachers had negative opinions regarding proof. Most of the participants regard proof as an unnecessary and meaningless activity that must be memorized. This result supports the findings of the studies in which most of the students had negative opinions against proof (Moralı et al., 2006). Among the pre-service teachers, only Bade has a positive opinion regarding proof. In her statements, Bade emphasizes that proofs reinforce her conceptual knowledge and increase retention. Bade's statement corresponds with the research result that argues that students' mathematical knowledge develops and matures via mathematical proofs (Kitcher, 1984).

In order to find an answer to the third question of the research, pre-service teachers were requested to prove again a theorem that they had proven before. It was found that most of the participants failed in performing proof. In view of this, it can be stated that the pre-service teachers' proof skills are at a considerably low level. It was found that only one participant (Bade) succeeded in performing proof and developed a proof strategy for herself. This method reminds us of Polya's (Polya, 1945) problem solving steps. This result supports the research result of Furinghetti & Morselli (2009) arguing that there is a strong relationship between proof process and problem solving and this process is composed of Polya's steps of understanding the problem, devising a plan, carrying out the plan and looking back. This result, which was obtained in the study, shows parallelism with the research results, arguing that the students failed in performing proof (Arslan, 2007; Arslan & Yıldız, 2010; Coşkun, 2009; Moore, 1994). Furthermore, it was found that the majority of pre-service teachers were not able to perform the proof although the theorem that they were required to prove was a theorem that they had proven before. This result supports Weber's (2001) research result arguing, "The fact that the pre-service teachers remember a theorem or a concept does not guarantee that they will perform the proof."

In order to reach the aim of the research, the answers to the research questions were evaluated with a holistic approach, and an attempt was made to discover the patterns among the results. The participant who had an adequate level of conceptual knowledge on the convergence of sequence succeeded in performing proof whereas other participants with an inadequate level of conceptual knowledge on the convergence of sequence failed in performing proof. This condition calls to mind that there is a close relationship between conceptual knowledge and proof skill. This result corresponds with the research results arguing that lack of conceptual knowledge constitutes a difficulty in performing proof (Gibson, 1998). When the relationship between opinions regarding proof and proof skill was taken into account, it was observed that the participant who had a positive opinion regarding proof succeeded in performing proof whereas other pre-service teachers with negative opinions regarding proof failed in performing proof. Therefore, it can be stated that opinions regarding proof have a connection with proof skill. This result, which was obtained in the study, corresponds with the research results arguing that students' opinions regarding proof affect their activities related with proof (Furinghetti & Morselli, 2009; Weber, 2004). Lastly, when the participants' academic success in the related course, their opinions regarding proof and their proof skills were examined, it could be said that students' course success was not reflected on their proof skills and opinions regarding proof. Although the participants of the research were selected among the pre-service teachers who successfully passed the related course, the majority of the participants

failed in performing proof. This result corresponds with the fact that proof is a subject which is accepted with resistance even by successful students (Weber, 2004).

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